General Relativity II
(Cosmology)

Lecture 1

- Organization of the course
- Brief history of the Universe and plan of the course
- FRW spacetime
- Friedman equations (intro)

- Organization:

Grading: written exam.

Homework problems: distributed after the lecture, discussed at the next recitation

Moodle: forum for questions

Literature: Prof. Shaposhnikov notes,

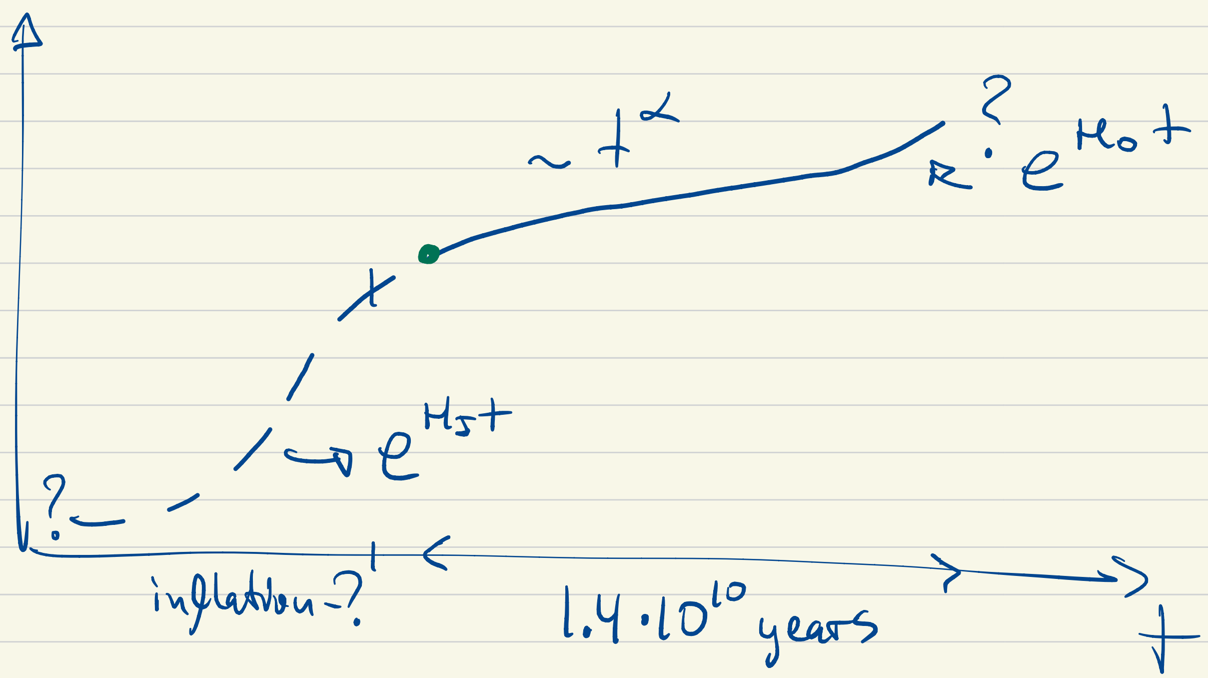
Books: Landau–Lifshitz vol II
Kolb – Turner
Weinberg (2007)
Gorbunov – Rubakov (2 parts)

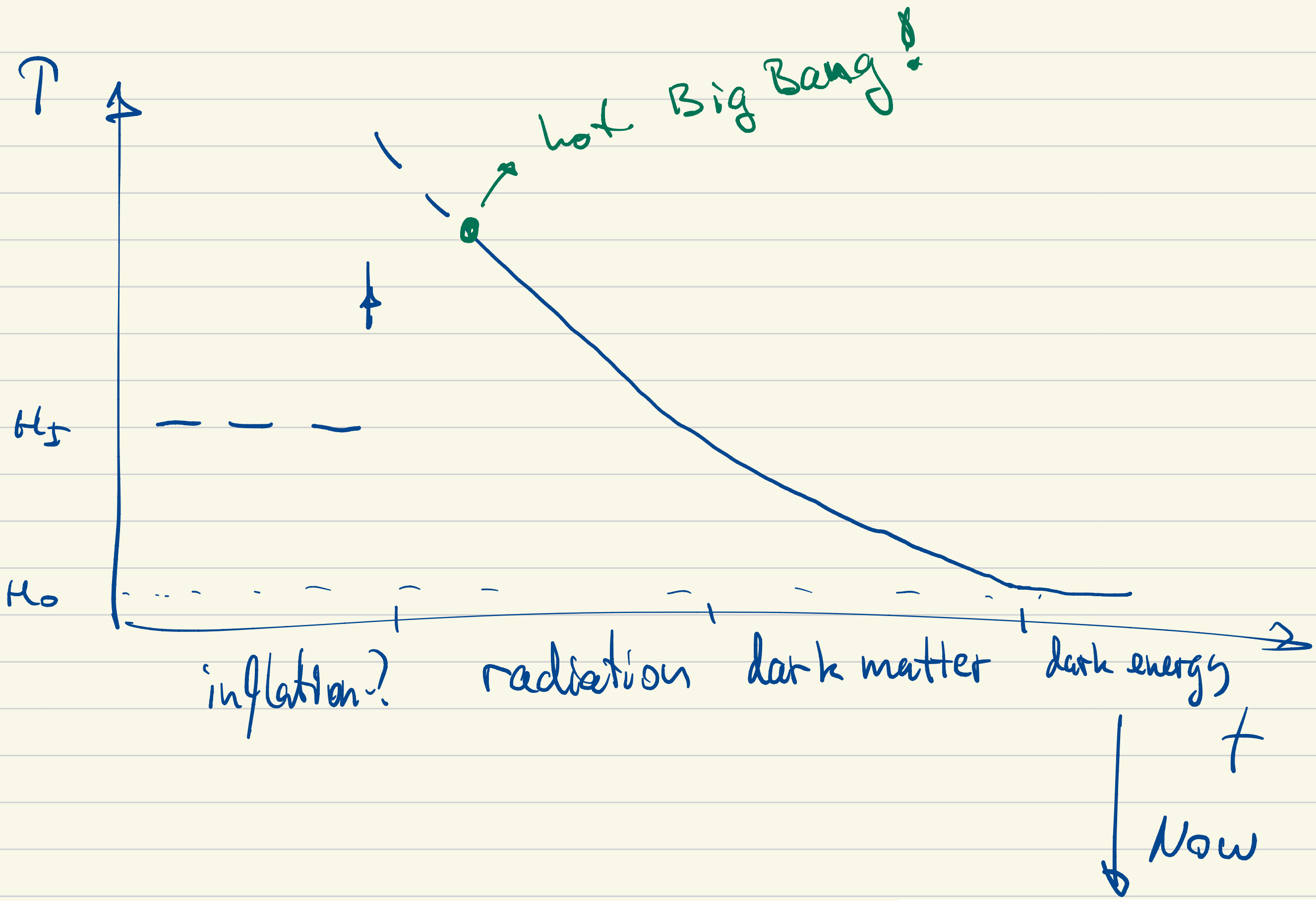
Brief history of the Universe and plan of the course

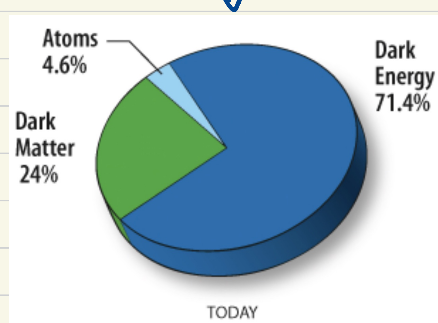
Basic facts about the Universe

- On very large scales (on average) Universe is isotropic, homogeneous and spacially flat.
- It has been expanding [distances between distant objects grow]

$a(t)$ ~ scale factor ("size" of the U)







- Size of observable U now: $5 \cdot 10^3$ Mpc
- Homogeneous on scales $\gtrsim 10$ Mpc

$1\,\text{pc} \approx 3\ \text{l.y.} \simeq 3 \cdot 10^{16}\ \text{m}$

Earth – Sun: $1.5 \cdot 10^{11}\ \text{m}$

Galaxy: $5 \cdot 10^4$ l.y.

Timeline:

1 MeV	50 keV	0.8 eV	0.26 eV	stars formation	4.4 K	2.7 K	T
1 s	5 min	$5 \cdot 10^4$ y	$3.7 \cdot 10^5$ y	10^8 y	$7.7 \cdot 10^9$ y	$1.4 \cdot 10^{10}$ y	t
BBN		matter dom.	recombination		transition c.c. dom.	now	

$$1\,eV = 1{,}2 \cdot 10^4\,K$$

Plan of the course:

- FRW metric, homogeneous universe, distances and horizons, cosmological epochs, ΛCDM
- Thermal history, evolution of matter and radiation, CMB, BBN (briefly Baryogenesis)
- Dark matter (evidence for it and most popular candidates)
- Theory of cosmological perturbations: CMB and growth of structure
- Inflation

FRW spacetime

- Universe is homogeneous and isotropic on large scales (that we still can observe)
- Based on these symmetries we can determine the spacial metric g_{ij} which describes geometry at given time (timeslice).

$$R, \qquad R_{ij} = R\, g_{ij} \cdot \frac{1}{3}$$

$$R_{ijkl} = c \cdot R \cdot [g_{ik} g_{jl} - g_{il} g_{jk}]$$

$$2c = \frac{1}{3} \Rightarrow c = \frac{1}{6}$$

- There are three cases:

$$R > 0 \qquad R = 0 \qquad R < 0$$

$R=0$ is just the three-plane R^3

$R=0$ is the three-sphere S^3

defined by the equation

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = a^2$$

clearly this equation is homogeneous and isotropic.

Let us use the following coordinates on the sphere:

$$x_1 = r \sin\theta \cos\varphi$$

$$x_2 = r \sin\theta \sin\varphi$$

$$x_3 = r \cos\theta$$

$$x_4^2 = a^2 - r^2 \Rightarrow x_4 = \sqrt{a^2-r^2}$$

$$dx_1^2 + dx_2^2 + dx_3^2 = dr^2 + r^2 d\Omega_2^2$$

$$\downarrow$$

$$\underbrace{d\theta^2 + \sin^2\theta \, d\varphi^2}_{S^2}$$

$$dx_4^2 = \frac{dr^2 \cdot r^2}{a^2 - r^2}$$

$$\Rightarrow ds^2 = \frac{a^2 dr^2}{a^2 - r^2} + r^2 d\Omega_2^2$$

Curvature of the sphere is proportional to a^{-2}: $\quad R \sim a^{-2}$

Consequently to get a negatively curved space we can try

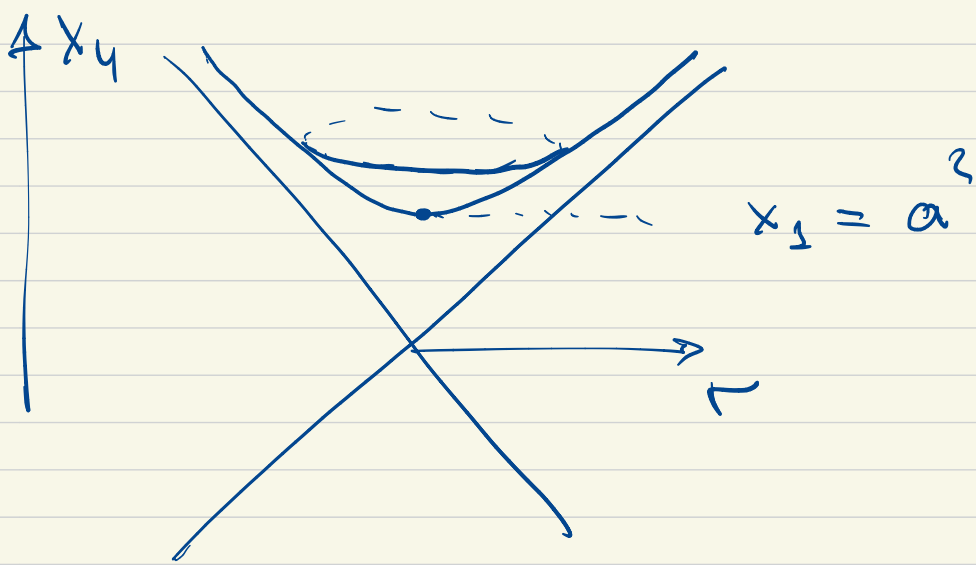
$$a^2 \to -a^2$$

$$ds^2 = \frac{a^2 dr^2}{a^2 + r^2} + r^2 d\Omega_2^2$$

This is called a hyperbolic space, also known as Euclidean Anti-de Sitter space.

Embedding?

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 = -a^2$$



$$r^2 = x_4^2 - a^2$$

- There is a unified description of all the metrics:

$$ds^2 = a^2 \left(\frac{dr^2}{1 - kr} + r^2 d\Omega_2^2 \right)$$

R^3 in sph. coord.

$k = 1, 0, -1$.

- Now we go back to describing the full four-dimensional spacetime geometry: this is done simply by adding the time coordinate:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\bar{r}^2}{1-K\bar{r}^2} + \bar{r}^2 d\Omega^2\right)$$

$a(t)$ so far is an arbitrary function of time – scale factor

$(-+++)$ metric.

- This is the metric of

Friedmann–Lemaitre–Robertson–Walker

it has the same symmetry properties (for generic $a(t)$)

- One can consider spaces with locally the same metric, but non-trivial topology.

Friedmann Equations

- On large (observable) scales the universe is well-approximated by the FRW metric. In the first part of the course we will study this homogeneous approximation.

- Our next goal will be to determine the function $a(t)$ from the Einstein equations.

- Let us remember them: